```
restart;
#Digits:=40;
with(LinearAlgebra) :
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Text S1

This document evaluates the derived potential dV/dAx

Ideal integrals

$$SSSS := (Rx, Ry, Rz, Tx, Ty, Tz) \rightarrow \frac{1}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)};$$

$$(Rx, Ry, Rz, Tx, Ty, Tz) \rightarrow \frac{1}{\sqrt{(Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2}} \quad (1.1)$$

$$SSPOPO := (Rx, Ry, Rz, Tx, Ty, Tz, D2) \rightarrow SSSS(Rx, Ry, Rz, Tx, Ty, Tz) + \frac{1}{2}\left(\frac{1}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2+(2\cdot D2)^2\right)} - SSSS(Rx, Ry, Rz, Tx, Ty, Tz)\right);$$

$$(Rx, Ry, Rz, Tx, Ty, Tz, D2) \rightarrow \frac{1}{2}\,SSSS(Rx, Ry, Rz, Tx, Ty, Tz) + \frac{1}{2\sqrt{(Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2+4\,D2^2}} \quad (1.2)$$

$$SSPZPZ := (Rx, Ry, Rz, Tx, Ty, Tz, D2) \rightarrow SSSS(Rx, Ry, Rz, Tx, Ty, Tz) + \frac{1}{4}\left(\frac{1}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)+2\,D2} + \frac{1}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)-2\,D2}\right) - \frac{1}{2}\cdot SSSS(Rx, Ry, Rz, Tx, Ty, Tz);$$

$$(Rx, Ry, Rz, Tx, Ty, Tz, D2) \rightarrow \frac{1}{2}\,SSSS(Rx, Ry, Rz, Tx, Ty, Tz) + \frac{1}{4\left(\sqrt{(Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2}+2\,D2\right)} + \frac{1}{4\left(\sqrt{(Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2}-2\,D2\right)} \quad (1.3)$$

$$SSSPZ := (Rx, Ry, Rz, Tx, Ty, Tz, D1) \rightarrow \frac{1}{2}\cdot\left(\frac{1}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)+\mathrm{D1}} - \frac{1}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)-D1}\right);$$

$$(Rx, Ry, Rz, Tx, Ty, Tz, D1) \rightarrow \frac{1}{2\left(\sqrt{(Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2}+D1\right)} \quad (1.4)$$

$$-\frac{1}{2\left(\sqrt{(Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2}-D1\right)}$$

Projection operators

$$X_1 := (Rx, Ry, Rz, Tx, Ty, Tz) \to \frac{Rx-Tx}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)} :$$

$X_1(Rx, Ry, Rz, Tx, Ty, Tz) :$

$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2=R2, \%\right) :$
$factor(\%);$

$$\frac{Rx-Tx}{\sqrt{R2}} \tag{2.1}$$

$$X_2 := (Rx, Ry, Rz, Tx, Ty, Tz) \to \frac{Ry-Ty}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)} :$$

$X_2(Rx, Ry, Rz, Tx, Ty, Tz) :$

$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2=R2, \%\right) :$
$factor(\%);$

$$\frac{Ry-Ty}{\sqrt{R2}} \tag{2.2}$$

$$X_3 := (Rx, Ry, Rz, Tx, Ty, Tz) \to \frac{Rz-Tz}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)} :$$

$X_3(Rx, Ry, Rz, Tx, Ty, Tz) :$

$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2=R2, \%\right) :$
$factor(\%);$

$$\frac{Rz-Tz}{\sqrt{R2}} \tag{2.3}$$

$$Y_1 := (Rx, Ry, Rz, Tx, Ty, Tz) \to -\frac{Ry-Ty}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2\right)} :$$

$Y_1(Rx, Ry, Rz, Tx, Ty, Tz) :$

$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2=R2, \%\right) :$
$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2=RY2, \%\right) :$
$factor(\%);$

$$-\frac{Ry-Ty}{\sqrt{RY2}} \tag{2.4}$$

$$Y_2 := (Rx, Ry, Rz, Tx, Ty, Tz) \to \frac{Rx-Tx}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2\right)} :$$

$Y_2(Rx, Ry, Rz, Tx, Ty, Tz) :$

$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2=R2, \%\right) :$
$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2=RY2, \%\right) :$
$factor(\%);$

$$\frac{Rx - Tx}{\sqrt{RY2}} \tag{2.5}$$

$Z_1 := (Rx, Ry, Rz, Tx, Ty, Tz) \rightarrow$

$$-\frac{(Rx - Tx)\cdot(Rz - Tz)}{\text{sqrt}\left((Rx - Tx)^2 + (Ry - Ty)^2 + (Rz - Tz)^2\right)\cdot\text{sqrt}\left((Rx - Tx)^2 + (Ry - Ty)^2\right)} :$$

$Z_1(Rx, Ry, Rz, Tx, Ty, Tz) :$

$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2 = R2, \%\right) :$

$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 = RY2, \%\right) :$

$factor(\%);$

$$-\frac{(Rx - Tx)\,(Rz - Tz)}{\sqrt{R2}\,\sqrt{RY2}} \tag{2.6}$$

$Z_2 := (Rx, Ry, Rz, Tx, Ty, Tz) \rightarrow$

$$-\frac{(Ry - Ty)\cdot(Rz - Tz)}{\text{sqrt}\left((Rx - Tx)^2 + (Ry - Ty)^2 + (Rz - Tz)^2\right)\cdot\text{sqrt}\left((Rx - Tx)^2 + (Ry - Ty)^2\right)} :$$

$Z_2(Rx, Ry, Rz, Tx, Ty, Tz) :$

$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2 = R2, \%\right) :$

$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 = RY2, \%\right) :$

$factor(\%);$

$$-\frac{(Ry - Ty)\,(Rz - Tz)}{\sqrt{R2}\,\sqrt{RY2}} \tag{2.7}$$

$Z_3 := (Rx, Ry, Rz, Tx, Ty, Tz)$

$$\rightarrow \frac{(Rx - Tx)^2 + (Ry - Ty)^2}{\text{sqrt}\left((Rx - Tx)^2 + (Ry - Ty)^2 + (Rz - Tz)^2\right)\cdot\text{sqrt}\left((Rx - Tx)^2 + (Ry - Ty)^2\right)} :$$

$Z_3(Rx, Ry, Rz, Tx, Ty, Tz) :$

$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2 = R2, \%\right) :$

$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 = RY2, \%\right) :$

$factor(\%);$

$$\frac{Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2}{\sqrt{R2}\,\sqrt{RY2}} \tag{2.8}$$

Derived ideal integrals

Here the simple derivative of the integrals are evaluated.

1) First we have the (ss|ss) integrals

$diff\,(SSSS(Rx, Ry, Rz, Tx, Ty, Tz), Rx) :$

$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2 = R2, \%\right) :$

$factor(\%);$

$$-\frac{Rx - Tx}{R2^{3/2}} \tag{3.1}$$

$diff\,(SSSS(Rx, Ry, Rz, Tx, Ty, Tz), Ry) :$

$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2 = R2, \%\right) :$

```
factor(%);
```

$$-\frac{Ry-Ty}{R2^{3/2}} \tag{3.2}$$

```
diff(SSSS(Rx, Ry, Rz, Tx, Ty, Tz), Rz):
subs(Rx^2 - 2 Rx Tx + Tx^2 + Ry^2 - 2 Ry Ty + Ty^2 + Rz^2 - 2 Rz Tz + Tz^2 = R2, %):
factor(%);
```

$$-\frac{Rz-Tz}{R2^{3/2}} \tag{3.3}$$

2) Second we have the (ss|p_op_o) integrals

```
diff(SSPOPO(Rx, Ry, Rz, Tx, Ty, Tz, D2), Rx);
```

$$-\frac{1}{4}\,\frac{2\,Rx-2\,Tx}{\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2\right)^{3/2}} \tag{3.4}$$

$$-\frac{1}{4}\,\frac{2\,Rx-2\,Tx}{\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2+4\,D2^2\right)^{3/2}}$$

(3.5)

(3.6)

```
diff(SSPOPO(Rx, Ry, Rz, Tx, Ty, Tz, D2), Ry);
```

$$-\frac{1}{4}\,\frac{2\,Ry-2\,Ty}{\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2\right)^{3/2}} \tag{3.7}$$

$$-\frac{1}{4}\,\frac{2\,Ry-2\,Ty}{\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2+4\,D2^2\right)^{3/2}}$$

(3.8)

(3.9)

```
diff(SSPOPO(Rx, Ry, Rz, Tx, Ty, Tz, D2), Rz);
```

$$-\frac{1}{4}\,\frac{2\,Rz-2\,Tz}{\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2\right)^{3/2}} \tag{3.10}$$

$$-\frac{1}{4}\,\frac{2\,Rz-2\,Tz}{\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2+4\,D2^2\right)^{3/2}}$$

(3.11)

(3.12)

3) Third we have the (ss|p_sp_s) integrals

```
diff(SSPZPZ(Rx, Ry, Rz, Tx, Ty, Tz, D2), Rx):
subs(Rx^2 - 2 Rx Tx + Tx^2 + Ry^2 - 2 Ry Ty + Ty^2 + Rz^2 - 2 Rz Tz + Tz^2 = R2, %):
factor(%);
```

$$-\frac{(Rx-Tx)\left(R2^2-2\,R2\,D2^2+8\,D2^4\right)}{R2^{3/2}\left(\sqrt{R2}+2\,D2\right)^2\left(-\sqrt{R2}+2\,D2\right)^2} \tag{3.13}$$

```
diff(SSPZPZ(Rx, Ry, Rz, Tx, Ty, Tz, D2), Ry):
subs(Rx^2 - 2 Rx Tx + Tx^2 + Ry^2 - 2 Ry Ty + Ty^2 + Rz^2 - 2 Rz Tz + Tz^2 = R2, %):
factor(%);
```

$$-\frac{(Ry-Ty)\left(R2^2-2\,R2\,D2^2+8\,D2^4\right)}{R2^{3/2}\left(\sqrt{R2}+2\,D2\right)^2\left(-\sqrt{R2}+2\,D2\right)^2} \tag{3.14}$$

```
diff(SSPZPZ(Rx, Ry, Rz, Tx, Ty, Tz, D2), Rz):
subs(Rx^2 - 2*Rx*Tx + Tx^2 + Ry^2 - 2*Ry*Ty + Ty^2 + Rz^2 - 2*Rz*Tz + Tz^2 = R2, %):
factor(%);
```

$$-\frac{(Rz-Tz)\left(R2^2-2\,R2\,D2^2+8\,D2^4\right)}{R2^{3/2}\left(\sqrt{R2}+2\,D2\right)^2\left(-\sqrt{R2}+2\,D2\right)^2} \tag{3.15}$$

4) Finally we have the (ss|sp_s) integrals

```
diff(SSSPZ(Rx, Ry, Rz, Tx, Ty, Tz, D1), Rx):
subs(Rx^2 - 2*Rx*Tx + Tx^2 + Ry^2 - 2*Ry*Ty + Ty^2 + Rz^2 - 2*Rz*Tz + Tz^2 = R2, %):
factor(%);
```

$$\frac{2\,(Rx-Tx)\,D1}{\left(\sqrt{R2}+D1\right)^2\left(-\sqrt{R2}+D1\right)^2} \tag{3.16}$$

```
diff(SSSPZ(Rx, Ry, Rz, Tx, Ty, Tz, D1), Ry):
subs(Rx^2 - 2*Rx*Tx + Tx^2 + Ry^2 - 2*Ry*Ty + Ty^2 + Rz^2 - 2*Rz*Tz + Tz^2 = R2, %):
factor(%);
```

$$\frac{2\,(Ry-Ty)\,D1}{\left(\sqrt{R2}+D1\right)^2\left(-\sqrt{R2}+D1\right)^2} \tag{3.17}$$

```
diff(SSSPZ(Rx, Ry, Rz, Tx, Ty, Tz, D1), Rz):
subs(Rx^2 - 2*Rx*Tx + Tx^2 + Ry^2 - 2*Ry*Ty + Ty^2 + Rz^2 - 2*Rz*Tz + Tz^2 = R2, %):
factor(%);
```

$$\frac{2\,(Rz-Tz)\,D1}{\left(\sqrt{R2}+D1\right)^2\left(-\sqrt{R2}+D1\right)^2} \tag{3.18}$$

Derived projection operators

The projection operators are

X = R - T

normalization of X is sqrt(1-X[3]*X[3])

Y = [-X[2], X[1], 0]

Z = X \times Y

In the manuscript, they are R = X, u = Y and w = Z

Derivatives of projection operator X

x[1]

```
diff( (Rx - Tx)/sqrt((Rx - Tx)^2 + (Ry - Ty)^2 + (Rz - Tz)^2), Rx):
subs(Rx^2 - 2*Rx*Tx + Tx^2 + Ry^2 - 2*Ry*Ty + Ty^2 + Rz^2 - 2*Rz*Tz + Tz^2 = R2, %):
```

$$factor(\%);$$

$$\frac{R2 - Rx^2 + 2\,Rx\,Tx - Tx^2}{R2^{3/2}} \tag{4.1.1}$$

$$diff\left(\frac{Rx - Tx}{\mathrm{sqrt}\left((Rx - Tx)^2 + (Ry - Ty)^2 + (Rz - Tz)^2\right)}, Ry\right):$$
$$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2 = R2, \%\right):$$
$$factor(\%);$$

$$-\frac{(Rx - Tx)\,(Ry - Ty)}{R2^{3/2}} \tag{4.1.2}$$

$$diff\left(\frac{Rx - Tx}{\mathrm{sqrt}\left((Rx - Tx)^2 + (Ry - Ty)^2 + (Rz - Tz)^2\right)}, Rz\right):$$
$$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2 = R2, \%\right):$$
$$factor(\%);$$

$$-\frac{(Rx - Tx)\,(Rz - Tz)}{R2^{3/2}} \tag{4.1.3}$$

x[2]

$$diff\left(\frac{Ry - Ty}{\mathrm{sqrt}\left((Rx - Tx)^2 + (Ry - Ty)^2 + (Rz - Tz)^2\right)}, Rx\right):$$
$$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2 = R2, \%\right):$$
$$factor(\%);$$

$$-\frac{(Rx - Tx)\,(Ry - Ty)}{R2^{3/2}} \tag{4.1.4}$$

$$diff\left(\frac{Ry - Ty}{\mathrm{sqrt}\left((Rx - Tx)^2 + (Ry - Ty)^2 + (Rz - Tz)^2\right)}, Ry\right):$$
$$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2 = R2, \%\right):$$
$$factor(\%);$$

$$\frac{R2 - Ry^2 + 2\,Ry\,Ty - Ty^2}{R2^{3/2}} \tag{4.1.5}$$

$$diff\left(\frac{Ry - Ty}{\mathrm{sqrt}\left((Rx - Tx)^2 + (Ry - Ty)^2 + (Rz - Tz)^2\right)}, Rz\right):$$
$$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2 = R2, \%\right):$$
$$factor(\%);$$

$$-\frac{(Ry - Ty)\,(Rz - Tz)}{R2^{3/2}} \tag{4.1.6}$$

x[3]

$$diff\left(\frac{Rz - Tz}{\mathrm{sqrt}\left((Rx - Tx)^2 + (Ry - Ty)^2 + (Rz - Tz)^2\right)}, Rx\right):$$
$$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2 = R2, \%\right):$$
$$factor(\%);$$

$$-\frac{(Rx - Tx)\,(Rz - Tz)}{R2^{3/2}} \tag{4.1.7}$$

$$diff\left(\frac{Rz-Tz}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)},Ry\right):$$
$$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2=R2,\%\right):$$
$$factor(\%);$$

$$-\frac{(Ry-Ty)\,(Rz-Tz)}{R2^{3/2}} \qquad \textbf{(4.1.8)}$$

$$diff\left(\frac{Rz-Tz}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)},Rz\right):$$
$$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2=R2,\%\right):$$
$$factor(\%);$$

$$\frac{R2-Rz^2+2\,Rz\,Tz-Tz^2}{R2^{3/2}} \qquad \textbf{(4.1.9)}$$

x[1]*x[1]

$$diff\left(\left(\frac{Rx-Tx}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)}\right)^2,Rx\right):$$
$$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2=R2,\%\right):$$
$$factor(\%);$$

$$\frac{2\,(Rx-Tx)\,\left(R2-Rx^2+2\,Rx\,Tx-Tx^2\right)}{R2^2} \qquad \textbf{(4.1.10)}$$

$$diff\left(\left(\frac{Rx-Tx}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)}\right)^2,Ry\right):$$
$$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2=R2,\%\right):$$
$$factor(\%);$$

$$-\frac{2\,(Rx-Tx)^2\,(Ry-Ty)}{R2^2} \qquad \textbf{(4.1.11)}$$

$$diff\left(\left(\frac{Rx-Tx}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)}\right)^2,Rz\right):$$
$$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2=R2,\%\right):$$
$$factor(\%);$$

$$-\frac{2\,(Rx-Tx)^2\,(Rz-Tz)}{R2^2} \qquad \textbf{(4.1.12)}$$

x[2]*x[2]

$$diff\left(\left(\frac{Ry-Ty}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)}\right)^2,Rx\right):$$
$$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2=R2,\%\right):$$
$$factor(\%);$$

$$-\frac{2\,(Ry-Ty)^2\,(Rx-Tx)}{R2^2} \qquad \textbf{(4.1.13)}$$

$$diff\left(\left(\frac{Ry-Ty}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)}\right)^2, Ry\right):$$
$$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2=R2, \%\right):$$
$$factor(\%);$$

$$\frac{2\,(Ry-Ty)\left(R2-Ry^2+2\,Ry\,Ty-Ty^2\right)}{R2^2} \tag{4.1.14}$$

$$diff\left(\left(\frac{Ry-Ty}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)}\right)^2, Rz\right):$$
$$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2=R2, \%\right):$$
$$factor(\%);$$

$$-\frac{2\,(Ry-Ty)^2\,(Rz-Tz)}{R2^2} \tag{4.1.15}$$

x[3]*x[3]

$$diff\left(\left(\frac{Rz-Tz}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)}\right)^2, Rx\right):$$
$$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2=R2, \%\right):$$
$$factor(\%);$$

$$-\frac{2\,(Rz-Tz)^2\,(Rx-Tx)}{R2^2} \tag{4.1.16}$$

$$diff\left(\left(\frac{Rz-Tz}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)}\right)^2, Ry\right):$$
$$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2=R2, \%\right):$$
$$factor(\%);$$

$$-\frac{2\,(Rz-Tz)^2\,(Ry-Ty)}{R2^2} \tag{4.1.17}$$

$$diff\left(\left(\frac{Rz-Tz}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)}\right)^2, Rz\right):$$
$$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2=R2, \%\right):$$
$$factor(\%);$$

$$\frac{2\,(Rz-Tz)\left(R2-Rz^2+2\,Rz\,Tz-Tz^2\right)}{R2^2} \tag{4.1.18}$$

x[2]*x[1]

$$diff\left(\frac{Ry-Ty}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)}\cdot\frac{Rx-Tx}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)}, Rx\right);$$

$$-\frac{(Ry-Ty)\,(Rx-Tx)\,(2\,Rx-2\,Tx)}{\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2\right)^2}+\frac{Ry-Ty}{Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2} \tag{4.1.19}$$

$$diff\left(\frac{Ry-Ty}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)}\cdot\frac{Rx-Tx}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)},Ry\right);$$

$$\frac{Rx-Tx}{Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2}-\frac{(Ry-Ty)\,(Rx-Tx)\,(2\,Ry-2\,Ty)}{\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2\right)^2} \tag{4.1.20}$$

$$diff\left(\frac{Ry-Ty}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)}\cdot\frac{Rx-Tx}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)},Rz\right);$$

$$-\frac{(Ry-Ty)\,(Rx-Tx)\,(2\,Rz-2\,Tz)}{\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2\right)^2} \tag{4.1.21}$$

x[3]*x[1]

$$diff\left(\frac{Rz-Tz}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)}\cdot\frac{Rx-Tx}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)},Rx\right);$$

$$-\frac{(Rz-Tz)\,(Rx-Tx)\,(2\,Rx-2\,Tx)}{\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2\right)^2}+\frac{Rz-Tz}{Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2} \tag{4.1.22}$$

$$diff\left(\frac{Rz-Tz}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)}\cdot\frac{Rx-Tx}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)},Ry\right);$$

$$-\frac{(Rz-Tz)\,(Rx-Tx)\,(2\,Ry-2\,Ty)}{\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2\right)^2} \tag{4.1.23}$$

$$diff\left(\frac{Rz-Tz}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)}\cdot\frac{Rx-Tx}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)},Rz\right);$$

$$\frac{Rx-Tx}{Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2}-\frac{(Rz-Tz)\,(Rx-Tx)\,(2\,Rz-2\,Tz)}{\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2\right)^2} \tag{4.1.24}$$

x[3]*x[2]

$$diff\left(\frac{Rz-Tz}{\mathrm{sqrt}\left((Rx-Tx)^2+(Ry-Ty)^2+(Rz-Tz)^2\right)}\right.$$

$$\cdot \frac{Ry - Ty}{\mathrm{sqrt}\left((Rx - Tx)^2 + (Ry - Ty)^2 + (Rz - Tz)^2\right)}, Rx\Big);$$

$$-\frac{(Rz - Tz)\,(Ry - Ty)\,(2\,Rx - 2\,Tx)}{\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2\right)^2} \tag{4.1.25}$$

$$diff\Big(\frac{Rz - Tz}{\mathrm{sqrt}\left((Rx - Tx)^2 + (Ry - Ty)^2 + (Rz - Tz)^2\right)}$$

$$\cdot \frac{Ry - Ty}{\mathrm{sqrt}\left((Rx - Tx)^2 + (Ry - Ty)^2 + (Rz - Tz)^2\right)}, Ry\Big);$$

$$-\frac{(Rz - Tz)\,(Ry - Ty)\,(2\,Ry - 2\,Ty)}{\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2\right)^2} \tag{4.1.26}$$

$$+\frac{Rz - Tz}{Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2}$$

$$diff\Big(\frac{Rz - Tz}{\mathrm{sqrt}\left((Rx - Tx)^2 + (Ry - Ty)^2 + (Rz - Tz)^2\right)}$$

$$\cdot \frac{Ry - Ty}{\mathrm{sqrt}\left((Rx - Tx)^2 + (Ry - Ty)^2 + (Rz - Tz)^2\right)}, Rz\Big);$$

$$\frac{Ry - Ty}{Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2} \tag{4.1.27}$$

$$-\frac{(Rz - Tz)\,(Ry - Ty)\,(2\,Rz - 2\,Tz)}{\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2\right)^2}$$

Derivatives of projection operator Y

y[1]*y[1] = x[2]*x[2] / (1-x[3]*x[3])

$$Y11_x := (Rx, Ry, Rz, Tx, Ty, Tz) \to diff\left(Y_1(Rx, Ry, Rz, Tx, Ty, Tz) \cdot Y_1(Rx, Ry, Rz, Tx, Ty, Tz), Rx\right):$$

$$Y11_x(Rx, Ry, Rz, Tx, Ty, Tz);$$

$$-\frac{(Ry - Ty)^2\,(2\,Rx - 2\,Tx)}{\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2\right)^2} \tag{4.2.1}$$

$$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 = RY2, \%\right);$$

$$-\frac{(Ry - Ty)^2\,(2\,Rx - 2\,Tx)}{RY2^2} \tag{4.2.2}$$

$$Y11_y := (Rx, Ry, Rz, Tx, Ty, Tz) \to diff\left(Y_1(Rx, Ry, Rz, Tx, Ty, Tz) \cdot Y_1(Rx, Ry, Rz, Tx, Ty, Tz), Ry\right):$$

$$Y11_y(Rx, Ry, Rz, Tx, Ty, Tz);$$

$$\frac{2\,(Ry - Ty)}{Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2} \tag{4.2.3}$$

$$-\frac{(Ry - Ty)^2\,(2\,Ry - 2\,Ty)}{\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2\right)^2}$$

(4.2.4)

$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 = RY2, \%\right);$

$$\frac{2\,(Ry - Ty)}{RY2} - \frac{(Ry - Ty)^2\,(2\,Ry - 2\,Ty)}{RY2^2} \tag{4.2.5}$$

$Y11_z := (Rx, Ry, Rz, Tx, Ty, Tz) \rightarrow diff\left(Y_1(Rx, Ry, Rz, Tx, Ty, Tz) \cdot Y_1(Rx, Ry, Rz, Tx, Ty, Tz), Rz\right):$

$Y11_z(Rx, Ry, Rz, Tx, Ty, Tz);$

$$0 \tag{4.2.6}$$

(4.2.7)

$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 = RY2, \%\right);$

$$0 \tag{4.2.8}$$

y[2]*y[1] = -x[1]*x[2] / (1-x[3]*x[3])

$Y21_x := (Rx, Ry, Rz, Tx, Ty, Tz) \rightarrow diff\left(Y_2(Rx, Ry, Rz, Tx, Ty, Tz) \cdot Y_1(Rx, Ry, Rz, Tx, Ty, Tz), Rx\right):$

$Y21_x(Rx, Ry, Rz, Tx, Ty, Tz);$

$$-\frac{Ry - Ty}{Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2} + \frac{(Rx - Tx)\,(Ry - Ty)\,(2\,Rx - 2\,Tx)}{\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2\right)^2} \tag{4.2.9}$$

(4.2.10)

$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 = RY2, \%\right);$

$$-\frac{Ry - Ty}{RY2} + \frac{(Rx - Tx)\,(Ry - Ty)\,(2\,Rx - 2\,Tx)}{RY2^2} \tag{4.2.11}$$

$Y21_y := (Rx, Ry, Rz, Tx, Ty, Tz) \rightarrow diff\left(Y_2(Rx, Ry, Rz, Tx, Ty, Tz) \cdot Y_1(Rx, Ry, Rz, Tx, Ty, Tz), Ry\right):$

$Y21_y(Rx, Ry, Rz, Tx, Ty, Tz);$

$$\frac{(Rx - Tx)\,(Ry - Ty)\,(2\,Ry - 2\,Ty)}{\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2\right)^2} - \frac{Rx - Tx}{Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2} \tag{4.2.12}$$

(4.2.13)

$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 = RY2, \%\right);$

$$\frac{(Rx - Tx)\,(Ry - Ty)\,(2\,Ry - 2\,Ty)}{RY2^2} - \frac{Rx - Tx}{RY2} \tag{4.2.14}$$

$Y21_z := (Rx, Ry, Rz, Tx, Ty, Tz) \rightarrow diff\left(Y_2(Rx, Ry, Rz, Tx, Ty, Tz) \cdot Y_1(Rx, Ry, Rz, Tx, Ty, Tz), Rz\right):$

$Y21_z(Rx, Ry, Rz, Tx, Ty, Tz);$

$$0 \tag{4.2.15}$$

(4.2.16)

$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 = RY2, \%\right);$

$$0 \tag{4.2.17}$$

y[2]*y[2] = x[1]*x[1] / (1-x[3]*x[3])

$Y22_x := (Rx, Ry, Rz, Tx, Ty, Tz) \rightarrow diff\left(Y_2(Rx, Ry, Rz, Tx, Ty, Tz) \cdot Y_2(Rx, Ry, Rz, Tx, Ty, Tz), Rx\right):$

$Y22_x(Rx, Ry, Rz, Tx, Ty, Tz);$

$$\frac{2\,(Rx - Tx)}{Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2} - \frac{(Rx - Tx)^2\,(2\,Rx - 2\,Tx)}{\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2\right)^2} \tag{4.2.18}$$

$\#eval(\%, \{Rx = -16.692327595, Ry = 2.053546334, Rz = 4.935321773, Tx = -16.319510615, Ty = 3.200959013, Tz = 8.115683388, D2 = 0.512573800\});$

$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 = RY2, \%\right);$

$$\frac{2\,(Rx - Tx)}{RY2} - \frac{(Rx - Tx)^2\,(2\,Rx - 2\,Tx)}{RY2^2} \tag{4.2.19}$$

$Y22_y := (Rx, Ry, Rz, Tx, Ty, Tz) \rightarrow diff\left(Y_2(Rx, Ry, Rz, Tx, Ty, Tz) \cdot Y_2(Rx, Ry, Rz, Tx, Ty, Tz), Ry\right):$

$Y22_y(Rx, Ry, Rz, Tx, Ty, Tz);$

$$-\frac{(Rx - Tx)^2\,(2\,Ry - 2\,Ty)}{\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2\right)^2} \tag{4.2.20}$$

$\#eval(\%, \{Rx = -16.692327595, Ry = 2.053546334, Rz = 4.935321773, Tx = -16.319510615, Ty = 3.200959013, Tz = 8.115683388, D2 = 0.512573800\});$

$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 = RY2, \%\right);$

$$-\frac{(Rx - Tx)^2\,(2\,Ry - 2\,Ty)}{RY2^2} \tag{4.2.21}$$

$Y22_z := (Rx, Ry, Rz, Tx, Ty, Tz) \rightarrow diff\left(Y_2(Rx, Ry, Rz, Tx, Ty, Tz) \cdot Y_2(Rx, Ry, Rz, Tx, Ty, Tz), Rz\right):$

(4.2.22)

$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 = RY2, \%\right);$

$$(Rx, Ry, Rz, Tx, Ty, Tz) \rightarrow \frac{\partial}{\partial Rz}\left(Y_2(Rx, Ry, Rz, Tx, Ty, Tz)^2\right) \tag{4.2.23}$$

Derivatives of projection operator Z

z[1]*z[1]

$Z11_x := (Rx, Ry, Rz, Tx, Ty, Tz) \rightarrow diff\left(Z_1(Rx, Ry, Rz, Tx, Ty, Tz) \cdot Z_1(Rx, Ry, Rz, Tx, Ty, Tz), Rx\right):$

$Z11_x(Rx, Ry, Rz, Tx, Ty, Tz);$

$\left(2\,(Rx - Tx)\,(Rz - Tz)^2\right) / \left(\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2\right.\right.$

$$-2\,Rz\,Tz+Tz^2)\,(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2))-((Rx-Tx)^2\,(Rz-Tz)^2\,(2\,Rx-$$
$$+Ty^2))-((Rx-Tx)^2\,(Rz-Tz)^2\,(2\,Rx-2\,Tx))\Big/((Rx^2-2\,Rx\,Tx+Tx^2$$
$$+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2)\,(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2$$
$$-2\,Ry\,Ty+Ty^2)^2)$$

$\#eval(\%, \{Rx=-16.692327595, Ry=2.053546334, Rz=4.935321773, Tx=-16.319510615, Ty=3.200959013, Tz=8.115683388, D2=0.512573800\});$

$subs(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2=R2, \%):$

$subs(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2=RY2, \%);$

$$\frac{2\,(Rx-Tx)\,(Rz-Tz)^2}{R2\,RY2}-\frac{(Rx-Tx)^2\,(Rz-Tz)^2\,(2\,Rx-2\,Tx)}{R2^2\,RY2}$$
$$-\frac{(Rx-Tx)^2\,(Rz-Tz)^2\,(2\,Rx-2\,Tx)}{R2\,RY2^2} \qquad \textbf{(4.3.2)}$$

$Z11_y := (Rx, Ry, Rz, Tx, Ty, Tz) \to diff(Z_1(Rx, Ry, Rz, Tx, Ty, Tz)\cdot Z_1(Rx, Ry, Rz, Tx, Ty, Tz), Ry):$

$Z11_y(Rx, Ry, Rz, Tx, Ty, Tz);$

$$-((Rx-Tx)^2\,(Rz-Tz)^2\,(2\,Ry-2\,Ty))\Big/((Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2$$
$$+Rz^2-2\,Rz\,Tz+Tz^2)^2\,(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2))$$
$$-((Rx-Tx)^2\,(Rz-Tz)^2\,(2\,Ry-2\,Ty))\Big/((Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2$$
$$+Ty^2)^2) \qquad \textbf{(4.3.4)}$$

$subs(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2=R2, \%):$

$subs(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2=RY2, \%);$

$$-\frac{(Rx-Tx)^2\,(Rz-Tz)^2\,(2\,Ry-2\,Ty)}{R2^2\,RY2}-\frac{(Rx-Tx)^2\,(Rz-Tz)^2\,(2\,Ry-2\,Ty)}{R2\,RY2^2} \qquad \textbf{(4.3.5)}$$

$Z11_z := (Rx, Ry, Rz, Tx, Ty, Tz) \to diff(Z_1(Rx, Ry, Rz, Tx, Ty, Tz)\cdot Z_1(Rx, Ry, Rz, Tx, Ty, Tz), Rz):$

$Z11_z(Rx, Ry, Rz, Tx, Ty, Tz);$

$$(2\,(Rx-Tx)^2\,(Rz-Tz))\Big/((Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2$$
$$-2\,Rz\,Tz+Tz^2)\,(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2))-((Rx-Tx)^2\,(Rz-Tz)^2\,(2\,Rz-$$
$$+Ty^2))$$

$subs(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2=R2, \%):$

$subs(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2=RY2, \%);$

$$\frac{2\,(Rx-Tx)^2\,(Rz-Tz)}{R2\,RY2}-\frac{(Rx-Tx)^2\,(Rz-Tz)^2\,(2\,Rz-2\,Tz)}{R2^2\,RY2} \qquad \textbf{(4.3.7)}$$

z[2]*z[2]

$Z22_x := (Rx, Ry, Rz, Tx, Ty, Tz) \to diff(Z_2(Rx, Ry, Rz, Tx, Ty, Tz)\cdot Z_2(Rx, Ry, Rz, Tx, Ty, Tz),$

$Rx\big)$:

$Z22_x(Rx, Ry, Rz, Tx, Ty, Tz)$;

$$-\left((Ry-Ty)^2\,(Rz-Tz)^2\,(2\,Rx-2\,Tx)\right)\Big/\left(\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2\right)^2\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2\right)\right)-\left((Ry-Ty)^2\,(Rz-Tz)^2\,(2\,Rx-2\,Tx)\right)\Big/\left(\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2+Ty^2\right)^2\right)$$

$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2=R2,\%\right)$:

$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2=RY2,\%\right)$;

$$-\frac{(Ry-Ty)^2\,(Rz-Tz)^2\,(2\,Rx-2\,Tx)}{R2^2\,RY2}-\frac{(Ry-Ty)^2\,(Rz-Tz)^2\,(2\,Rx-2\,Tx)}{R2\,RY2^2} \qquad \textbf{(4.3.9)}$$

$Z22_y := (Rx, Ry, Rz, Tx, Ty, Tz) \to diff\left(Z_2(Rx, Ry, Rz, Tx, Ty, Tz)\cdot Z_2(Rx, Ry, Rz, Tx, Ty, Tz), Ry\right)$:

$Z22_y(Rx, Ry, Rz, Tx, Ty, Tz)$;

$$\left(2\,(Ry-Ty)\,(Rz-Tz)^2\right)\Big/\left(\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2\right)\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2\right)\right)-\left((Ry-Ty)^2\,(Rz-Tz)^2\,(2\,Ry-+Ty^2)\right)-\left((Ry-Ty)^2\,(Rz-Tz)^2\,(2\,Ry-2\,Ty)\right)\Big/\left(\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2\right)\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2\right)^2\right)$$

(4.3.11)

$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2=R2,\%\right)$:

$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2=RY2,\%\right)$;

$$\frac{2\,(Ry-Ty)\,(Rz-Tz)^2}{R2\,RY2}-\frac{(Ry-Ty)^2\,(Rz-Tz)^2\,(2\,Ry-2\,Ty)}{R2^2\,RY2}-\frac{(Ry-Ty)^2\,(Rz-Tz)^2\,(2\,Ry-2\,Ty)}{R2\,RY2^2} \qquad \textbf{(4.3.12)}$$

$Z22_z := (Rx, Ry, Rz, Tx, Ty, Tz) \to diff\left(Z_2(Rx, Ry, Rz, Tx, Ty, Tz)\cdot Z_2(Rx, Ry, Rz, Tx, Ty, Tz), Rz\right)$:

$Z22_z(Rx, Ry, Rz, Tx, Ty, Tz)$;

$$\left(2\,(Ry-Ty)^2\,(Rz-Tz)\right)\Big/\left(\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2\right)\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2\right)\right)-\left((Ry-Ty)^2\,(Rz-Tz)^2\,(2\,Rz-+Ty^2)\right)$$

$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2=R2,\%\right)$:

$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2=RY2,\%\right)$;

$$\frac{2\,(Ry-Ty)^2\,(Rz-Tz)}{R2\,RY2}-\frac{(Ry-Ty)^2\,(Rz-Tz)^2\,(2\,Rz-2\,Tz)}{R2^2\,RY2} \qquad \textbf{(4.3.14)}$$

z[3]*z[3] = 1-x[3]*x[3]

$$Z33_x := (Rx, Ry, Rz, Tx, Ty, Tz) \rightarrow diff\left(Z_3(Rx, Ry, Rz, Tx, Ty, Tz) \cdot Z_3(Rx, Ry, Rz, Tx, Ty, Tz), Rx\right):$$

$$Z33_x(Rx, Ry, Rz, Tx, Ty, Tz);$$

$$\left(2\left((Rx-Tx)^2+(Ry-Ty)^2\right)(2\,Rx-2\,Tx)\right)\Big/\left(\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2\right)\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2\right)\right)-\left(\left((Rx-Tx)^2+(Ry-Ty)^2\right)^2(2\,Rx-2\,Tx)\right)\Big/\left(\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+\right.\right.$$

$$\left.\left.-2\,Ry\,Ty+Ty^2\right)\right)-\left(\left((Rx-Tx)^2+(Ry-Ty)^2\right)^2(2\,Rx-2\,Tx)\right)\Big/\left(\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2\right)\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2\right)^2\right)$$

$$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2=R2, \%\right):$$

$$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2=RY2, \%\right):$$

$$subs\left((Rx-Tx)^2+(Ry-Ty)^2=RY2, \%\right);$$

$$\frac{2\,Rx-2\,Tx}{R2}-\frac{RY2\,(2\,Rx-2\,Tx)}{R2^2} \qquad \textbf{(4.3.16)}$$

$$Z33_y := (Rx, Ry, Rz, Tx, Ty, Tz) \rightarrow diff\left(Z_3(Rx, Ry, Rz, Tx, Ty, Tz) \cdot Z_3(Rx, Ry, Rz, Tx, Ty, Tz), Ry\right):$$

$$Z33_y(Rx, Ry, Rz, Tx, Ty, Tz);$$

$$\left(2\left((Rx-Tx)^2+(Ry-Ty)^2\right)(2\,Ry-2\,Ty)\right)\Big/\left(\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2\right)\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2\right)\right)-\left(\left((Rx-Tx)^2+(Ry-Ty)^2\right)^2(2\,Ry-2\,Ty)\right)\Big/\left(\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+\right.\right.$$

$$\left.\left.-2\,Ry\,Ty+Ty^2\right)\right)-\left(\left((Rx-Tx)^2+(Ry-Ty)^2\right)^2(2\,Ry-2\,Ty)\right)\Big/\left(\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2\right)\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2\right)^2\right)$$

$$\#eval(\%, \{Rx=-16.692327595, Ry=2.053546334, Rz=4.935321773, Tx=-16.319510615, Ty=3.200959013, Tz=8.115683388, D2=0.512573800\});$$

$$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2=R2, \%\right):$$

$$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2=RY2, \%\right):$$

$$subs\left((Rx-Tx)^2+(Ry-Ty)^2=RY2, \%\right);$$

$$\frac{2\,Ry-2\,Ty}{R2}-\frac{RY2\,(2\,Ry-2\,Ty)}{R2^2} \qquad \textbf{(4.3.18)}$$

$$Z33_z := (Rx, Ry, Rz, Tx, Ty, Tz) \rightarrow diff\left(Z_3(Rx, Ry, Rz, Tx, Ty, Tz) \cdot Z_3(Rx, Ry, Rz, Tx, Ty, Tz), Rz\right):$$

$$Z33_z(Rx, Ry, Rz, Tx, Ty, Tz);$$

$$-\left(\left((Rx-Tx)^2+(Ry-Ty)^2\right)^2(2\,Rz-2\,Tz)\right)\Big/\left(\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2\right)^2\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2\right)\right) \qquad \textbf{(4.3.19)}$$

$subs(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2 = R2, \%)$:
$subs(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 = RY2, \%)$:
$subs((Rx - Tx)^2 + (Ry - Ty)^2 = RY2, \%)$;

$$-\frac{RY2\,(2\,Rz - 2\,Tz)}{R2^2} \tag{4.3.20}$$

z[2]*z[1]

$Z21_x := (Rx, Ry, Rz, Tx, Ty, Tz) \rightarrow diff\left(Z_2(Rx, Ry, Rz, Tx, Ty, Tz) \cdot Z_1(Rx, Ry, Rz, Tx, Ty, Tz), Rx\right)$:
$Z21_x(Rx, Ry, Rz, Tx, Ty, Tz)$;

$$-\left((Ry - Ty)\,(Rz - Tz)^2\,(Rx - Tx)\,(2\,Rx - 2\,Tx)\right) \Big/ \left(\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2\right)^2 \left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2\right)\right) - \left((Ry - Ty)\,(Rz - Tz)^2\,(Rx - Tx)\,(2\,Rx - 2\,Tx)\right) \Big/ \left(\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2\right)\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2\right)^2\right) + \left((Ry - Ty)\,(Rz - Tz)^2\right) \Big/ \left(\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2\right)\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2\right)\right) \tag{4.3.21}$$

(4.3.22)

$subs(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2 = R2, \%)$:
$subs(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 = RY2, \%)$:
$subs((Rx - Tx)^2 + (Ry - Ty)^2 = RY2, \%)$;

$$-\frac{(Ry - Ty)\,(Rz - Tz)^2\,(Rx - Tx)\,(2\,Rx - 2\,Tx)}{R2^2\,RY2} - \frac{(Ry - Ty)\,(Rz - Tz)^2\,(Rx - Tx)\,(2\,Rx - 2\,Tx)}{R2\,RY2^2} + \frac{(Ry - Ty)\,(Rz - Tz)^2}{R2\,RY2} \tag{4.3.23}$$

$Z21_y := (Rx, Ry, Rz, Tx, Ty, Tz) \rightarrow diff\left(Z_2(Rx, Ry, Rz, Tx, Ty, Tz) \cdot Z_1(Rx, Ry, Rz, Tx, Ty, Tz), Ry\right)$:
$Z21_y(Rx, Ry, Rz, Tx, Ty, Tz)$;

$$\left((Rz - Tz)^2\,(Rx - Tx)\right) \Big/ \left(\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2\right)\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2\right)\right) - \left((Ry - Ty)\,(Rz - Tz)^2\,(Rx - Tx)\,(2\,Ry - 2\,Ty)\right) \Big/ \left(\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2\right)^2 \left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2\right)\right) - \left((Ry - Ty)\,(Rz - Tz)^2\,(Rx - Tx)\,(2\,Ry - 2\,Ty)\right) \Big/ \left(\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2\right)\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2\right)^2\right) \tag{4.3.24}$$

(4.3.25)

$subs(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2 = R2, \%)$:

$$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 = RY2,\ \%\right):$$
$$subs\left((Rx - Tx)^2 + (Ry - Ty)^2 = RY2,\ \%\right);$$

$$\frac{(Rz - Tz)^2\,(Rx - Tx)}{R2\,RY2} - \frac{(Ry - Ty)\,(Rz - Tz)^2\,(Rx - Tx)\,(2\,Ry - 2\,Ty)}{R2^2\,RY2} - \frac{(Ry - Ty)\,(Rz - Tz)^2\,(Rx - Tx)\,(2\,Ry - 2\,Ty)}{R2\,RY2^2} \tag{4.3.26}$$

$$Z21_z := (Rx, Ry, Rz, Tx, Ty, Tz) \to diff\left(Z_2(Rx, Ry, Rz, Tx, Ty, Tz) \cdot Z_1(Rx, Ry, Rz, Tx, Ty, Tz), Rz\right):$$
$$Z21_z(Rx, Ry, Rz, Tx, Ty, Tz);$$

$$(2\,(Ry - Ty)\,(Rz - Tz)\,(Rx - Tx))\Big/\left((Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2)\,(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2)\right) - \left((Ry - Ty)\,(Rz - Tz)^2\,(Rx - Tx)\,(2\,Rz - 2\,Tz)\right)\Big/\left((Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2)^2\,(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2)\right) \tag{4.3.27}$$

(4.3.28)

$$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2 = R2,\ \%\right):$$
$$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 = RY2,\ \%\right):$$
$$subs\left((Rx - Tx)^2 + (Ry - Ty)^2 = RY2,\ \%\right);$$

$$\frac{2\,(Ry - Ty)\,(Rz - Tz)\,(Rx - Tx)}{R2\,RY2} - \frac{(Ry - Ty)\,(Rz - Tz)^2\,(Rx - Tx)\,(2\,Rz - 2\,Tz)}{R2^2\,RY2} \tag{4.3.29}$$

z[3]*z[1] = -x[3]*x[1]

$$Z31_x := (Rx, Ry, Rz, Tx, Ty, Tz) \to diff\left(Z_3(Rx, Ry, Rz, Tx, Ty, Tz) \cdot Z_1(Rx, Ry, Rz, Tx, Ty, Tz), Rx\right):$$
$$Z31_x(Rx, Ry, Rz, Tx, Ty, Tz);$$

$$-((2\,Rx - 2\,Tx)\,(Rx - Tx)\,(Rz - Tz))\Big/\left((Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2)\,(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2)\right) + \left(\left((Rx - Tx)^2 + (Ry - Ty)^2\right)\,(Rx - Tx)\,(Rz - Tz)\,(2\,Rx - 2\,Tx)\right)\Big/\left((Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 + Ty^2)\right) + \left(\left((Rx - Tx)^2 + (Ry - Ty)^2\right)\,(Rx - Tx)\,(Rz - Tz)\,(2\,Rx - 2\,Tx)\right)\Big/\left((Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2)\,(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2)^2\right) - \left(\left((Rx - Tx)^2 + (Ry - Ty)^2\right)\,(Rz - Tz)\right)/\left(-2\,Ry\,Ty + Ty^2\right)\right)$$

(4.3.31)

$$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2 = R2,\ \%\right):$$
$$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 = RY2,\ \%\right):$$
$$subs\left((Rx - Tx)^2 + (Ry - Ty)^2 = RY2,\ \%\right);$$

$$\frac{(Rx-Tx)\,(Rz-Tz)\,(2\,Rx-2\,Tx)}{R2^2} \_ \frac{Rz-Tz}{R2} \tag{4.3.32}$$

$Z31_y := (Rx, Ry, Rz, Tx, Ty, Tz) \rightarrow diff\left(Z_3(Rx, Ry, Rz, Tx, Ty, Tz)\cdot Z_1(Rx, Ry, Rz, Tx, Ty, Tz), Ry\right)$:

$Z31_y(Rx, Ry, Rz, Tx, Ty, Tz)$;

$$-((2\,Ry-2\,Ty)\,(Rx-Tx)\,(Rz-Tz))\Big/\left((Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2)\,(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2)\right)+\left(\left((Rx-Tx)^2+(Ry-Ty)^2\right)\,(Rx-Tx)\,(Rz-Tz)\,(2\,Ry-2\,Ty)\right)\Big/\left((Rx^2-2\,Rx\,Tx+Tx^2+Ry^2+Ty^2)\right)+\left(\left((Rx-Tx)^2+(Ry-Ty)^2\right)\,(Rx-Tx)\,(Rz-Tz)\,(2\,Ry-2\,Ty)\right)\Big/\left((Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2)\,(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2)^2\right) \tag{4.3.34}$$

$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2=R2, \%\right)$:
$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2=RY2, \%\right)$:
$subs\left((Rx-Tx)^2+(Ry-Ty)^2=RY2, \%\right)$;

$$\frac{(Rx-Tx)\,(Rz-Tz)\,(2\,Ry-2\,Ty)}{R2^2} \tag{4.3.35}$$

$Z31_z := (Rx, Ry, Rz, Tx, Ty, Tz) \rightarrow diff\left(Z_3(Rx, Ry, Rz, Tx, Ty, Tz)\cdot Z_1(Rx, Ry, Rz, Tx, Ty, Tz), Rz\right)$:

$Z31_z(Rx, Ry, Rz, Tx, Ty, Tz)$;

$$\left(\left((Rx-Tx)^2+(Ry-Ty)^2\right)\,(Rx-Tx)\,(Rz-Tz)\,(2\,Rz-2\,Tz)\right)\Big/\left((Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2)^2\,(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2)\right)-\left(\left((Rx-Tx)^2+(Ry-Ty)^2\right)\,(Rx-Tx)\right)\Big/\left((Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2)\,(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2)\right) \tag{4.3.36}$$

$$\tag{4.3.37}$$

$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2=R2, \%\right)$:
$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2=RY2, \%\right)$:
$subs\left((Rx-Tx)^2+(Ry-Ty)^2=RY2, \%\right)$;

$$\frac{(Rx-Tx)\,(Rz-Tz)\,(2\,Rz-2\,Tz)}{R2^2} \_ \frac{Rx-Tx}{R2} \tag{4.3.38}$$

z[3]*z[2] = -x[3]*x[2]

$Z32_x := (Rx, Ry, Rz, Tx, Ty, Tz) \rightarrow diff\left(Z_3(Rx, Ry, Rz, Tx, Ty, Tz)\cdot Z_2(Rx, Ry, Rz, Tx, Ty, Tz), Rx\right)$:

$Z32_x(Rx, Ry, Rz, Tx, Ty, Tz)$;

$$-((2\,Rx-2\,Tx)\,(Ry-Ty)\,(Rz-Tz))\Big/\left((Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2)\,(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2)\right)$$

$$+\left(\left((Rx-Tx)^2+(Ry-Ty)^2\right)(Ry-Ty)(Rz-Tz)(2\,Rx-2\,Tx)\right)\Big/\left(\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2+Ty^2\right)\right)+\left(\left((Rx-Tx)^2+(Ry-Ty)^2\right)(Ry-Ty)(Rz-Tz)(2\,Rx-2\,Tx)\right)\Big/\left(\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2\right)\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2\right)^2\right) \tag{4.3.40}$$

$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2=R2,\%\right):$
$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2=RY2,\%\right):$
$subs\left((Rx-Tx)^2+(Ry-Ty)^2=RY2,\%\right);$

$$\frac{(Ry-Ty)(Rz-Tz)(2\,Rx-2\,Tx)}{R2^2} \tag{4.3.41}$$

$Z32_y := (Rx, Ry, Rz, Tx, Ty, Tz) \to diff\left(Z_3(Rx, Ry, Rz, Tx, Ty, Tz)\cdot Z_2(Rx, Ry, Rz, Tx, Ty, Tz), Ry\right):$
$Z32_y(Rx, Ry, Rz, Tx, Ty, Tz);$

$$-\left(\left(2\,Ry-2\,Ty\right)(Ry-Ty)(Rz-Tz)\right)\Big/\left(\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2\right)\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2\right)\right)+\left(\left((Rx-Tx)^2+(Ry-Ty)^2\right)(Ry-Ty)(Rz-Tz)(2\,Ry-2\,Ty)\right)\Big/\left(\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2+Ty^2\right)\right)+\left(\left((Rx-Tx)^2+(Ry-Ty)^2\right)(Ry-Ty)(Rz-Tz)(2\,Ry-2\,Ty)\right)\Big/\left(\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2\right)\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2\right)^2\right)-\left(\left((Rx-Tx)^2+(Ry-Ty)^2\right)(Rz-Tz)\right)/\left(\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2\right)\right) \tag{4.3.43}$$

$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2=R2,\%\right):$
$subs\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2=RY2,\%\right):$
$subs\left((Rx-Tx)^2+(Ry-Ty)^2=RY2,\%\right);$

$$\frac{(Ry-Ty)(Rz-Tz)(2\,Ry-2\,Ty)}{R2^2}-\frac{Rz-Tz}{R2} \tag{4.3.44}$$

$Z32_z := (Rx, Ry, Rz, Tx, Ty, Tz) \to diff\left(Z_3(Rx, Ry, Rz, Tx, Ty, Tz)\cdot Z_2(Rx, Ry, Rz, Tx, Ty, Tz), Rz\right):$
$Z32_z(Rx, Ry, Rz, Tx, Ty, Tz);$

$$\left(\left((Rx-Tx)^2+(Ry-Ty)^2\right)(Ry-Ty)(Rz-Tz)(2\,Rz-2\,Tz)\right)\Big/\left(\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2\right)^2\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2\right)\right)-\left(\left((Rx-Tx)^2+(Ry-Ty)^2\right)(Ry-Ty)\right)\Big/\left(\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2+Rz^2-2\,Rz\,Tz+Tz^2\right)\left(Rx^2-2\,Rx\,Tx+Tx^2+Ry^2-2\,Ry\,Ty+Ty^2\right)\right) \tag{4.3.45}$$

(4.3.46)

$$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 + Rz^2 - 2\,Rz\,Tz + Tz^2 = R2,\ \%\right):$$
$$subs\left(Rx^2 - 2\,Rx\,Tx + Tx^2 + Ry^2 - 2\,Ry\,Ty + Ty^2 = RY2,\ \%\right):$$
$$subs\left((Rx - Tx)^2 + (Ry - Ty)^2 = RY2,\ \%\right);$$

$$\frac{(Ry - Ty)\,(Rz - Tz)\,(2\,Rz - 2\,Tz)}{R2^2} \_ \frac{Ry - Ty}{R2} \qquad \textbf{(4.3.47)}$$