## Model Code for: Large-scale effects of migration and conflict in pre-agricultural groups: Insights from a dynamic model

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# Model code

This section presents the Matlab code used for the numerical simulation. The code was originally divided in three files, namely *MAIN.m*, *init.m*, and *shroedingerevol.m*.

#### Main (MAIN.m file)

 $\label{eq:lambda} \ensuremath{\scale{Structure}} \ensuremath$ 

%OUTPUT: % psi:=the evolved vector psi for all time in tspan % T:=the tspan % NH:= human densities in the cells at each time % NR:= densities of resources in the cells at each time %\*\*\*\*\*\*

## Initialization (init.m file)

%%Initial densities for Humans (dens\_initH) and Resources (dens\_initR)
for j=1:L
 dens\_initH(j)=rand(1);
 dens\_initR(j)=1-dens\_initH(1);
end

```
\% Ground state
phi0 = 1;
phi0(4^L)=0;
phi0=phi0';
% Oauli's Matrices
sp = [0 \ 1; 0 \ 0];
sz = [1 \ 0; 0 \ -1];
Id = [1 \ 0; 0 \ 1];
\%\ Fermionic\ Annihilation\ operators
p{2*L}=0;
for N=1:2*L;
  p\{N\}=sparse(sp);
  for n=N:2*L-1
    p{N}=kron(Id, p{N});
  \mathbf{end}
  for n=1:N-1
    p{N}=kron(p{N},sz);
  \mathbf{end}
\mathbf{end}
\% Initial condition: phiin
phiin=phi0;
phiin=sqrt(dens_initH(1))*p\{1\}'*phi0;
\texttt{phiin=phiin+sqrt} \left( \, \texttt{dens_initR} \left( 1 \right) \right) * \texttt{p} \{ 1 + L \} '* \, \texttt{phi0} ;
for j=2:L
  phiin=phiin+sqrt(dens_initH(j))*p{j}'*phi0;
  phiin=phiin+sqrt(dens_initR(j))*p{j+L}'*phi0;
end
phiin=phiin/norm(phiin);
%% Migration Matrix
MatrixR(L,L)=0;
MatrixR(1, 2:4) = [1 \ 1 \ 1];
MatrixR (2, [3:6]) = [1 \ 1 \ 1 \ 1];
MatrixR(3, [5 \ 6]) = [1 \ 1];
Matrix R(4, [5 \ 7 \ 8]) = [1 \ 1 \ 1];
MatrixR(5, [6 \ 7 \ 8 \ 9]) = [1 \ 1 \ 1 \ 1];
Matrix R(6, [8 \ 9]) = [1 \ 1];
Matrix R(7, 8) = [1];
MatrixR(8,9) = [1];
%Hamiltonian Operators
HH\{1,1\}=0;
for jpop=1:L-1
  for j=jpop+1:L
    HH{jpop,j}=(p{jpop}'*p{j}+p{j}'*p{jpop}); \ \% Migration \ term
  \mathbf{end}
\mathbf{end}
for jpop=1:L
  Hwh{jpop}=p{jpop}'*p{jpop}; %Free term for Humans
  Hwr{jpop}=p{jpop+L}'*p{jpop+L}; %Free term for Resources
  HR\{jpop\} = (p\{jpop\}'*p\{jpop+L\}+p\{jpop+L\}'*p\{jpop\});
      %Human-Resource interaction
end
```

# Runge Kutta numerical scheme for the Shroedinger equation (shroedingerevol.m file)

```
function [psi,T,NH,NR]=
shroedingerevol(L,p,phiin,matrixR,HH,HR,Hwh,Hwr,tspan)
```

```
% Runge Kutta numerical scheme for the Shroedinger equation
% Input parameters are set in the MAIN.m and init.m files
%OUTPUT:
% psi:=the evolved vector psi for all time in tspan
\% T := the tspan
% NH:= human densities in the cells at each time
% NR:= densities of resources in the cells at each time
%Runge-Kutta Scheme
options = odeset('RelTol', 1e-4, 'AbsTol', 1e-4*ones(length(phiin), 1))';
[T, psi] = ode45(@(t, z) secmem(L, p, matrixR, HH, HR, Hwh, Hwr, z, t),
  tspan, phiin, options);
%Evaluation of the densities in each cell
for t=1:length(tspan)
  for j=1:L
    NH(j,t)=norm(p\{j\}'*p\{j\}*psi(t,:)')^2;
    NR(j,t) = norm(p\{j+L\} * p\{j+L\} * psi(t,:) )^2;
  \mathbf{end}
\mathbf{end}
function jout=secmem(L,p,matrixR,HH,HR,Hwh,Hwr,z,t)
timescale = 1000;
paramscale=80;
for j=1:L
  NH(j) = norm(p\{j\}'*p\{j\}*z)^2;
  NR(j)=norm(p\{j+L\}, p\{j+L\}*z)^2;
  qq(j) = (NR(j)/NH(j));
  qq2(j) = (NH(j)/NR(j));
  ll(j) = sqrt(exp(-(qq(j)/0.35)^2)*(qq2(j)));
  rr(j) = sqrt(exp(-(qq2(j)/0.35)^2)*(qq(j)));
\mathbf{end}
omega(1:2*L) = [ones(1,L).*rr ones(1,L).*ll];
lambdaexH(L,L)=0;
for jpop=1:L-1
  for j=jpop+1:L
    lambdaexH(jpop, j) = (0.00 + ll(jpop) + ll(j)) * matrixR(jpop, j);
  \mathbf{end}
end
lambdaexR=ones(L,L);
mu(1:L)=ones(1,L).*(0.02+ll+rr);
H=sparse(4^{(L)}, 4^{(L)});
%%Inertial and interaction terms
for jpop=1:L
  H=H+omega(jpop)*Hwh{jpop}+omega(jpop+L)*Hwr{jpop}+
    mu(jpop)*(HR\{jpop\});
end
%%Migration terms
for jpop=1:L-1
  for j=jpop+1:L
    H=H+lambdaexH(jpop, j)*(HH\{jpop, j\});
  end
end
```