

## S1 File: Mathematical inference

2 For simplicity, we omitted matrix  $\mathbf{\Gamma}$  when analysing the dynamics of interac-  
tions, assuming that  $\mathbf{g}$  is now a vector of genotypic values, not genotypes, the  
4 same as  $\mathbf{\Gamma}\mathbf{g}$  in [1].

When individuals interact with social partners, the phenotype of the focal  
6 individual at time  $t + 1$  depends on the phenotypes of conspecifics at time  $t$  as

$$\begin{aligned} \mathbf{p}_i^{(t+1)} &= \mathbf{g}_i + \mathbf{\Psi} \sum_{j \neq i} \mathbf{p}_j^{(t)} \\ &= \mathbf{g}_i + \mathbf{\Psi} N \bar{\mathbf{p}}^t - \mathbf{\Psi} \mathbf{p}_i^t. \end{aligned} \quad (\text{S1})$$

We want to express each phenotype after  $n$  iterations as a function of the  
genotypes of social partners. The phenotype of the focal individual before in-  
teraction is

$$\mathbf{p}_i^{(0)} = \mathbf{g}_i \quad (\text{S2})$$

and the mean group phenotype before interaction

$$\bar{\mathbf{p}}^{(0)} = \bar{\mathbf{g}} \quad (\text{S3})$$

where  $\bar{\mathbf{g}} = \sum_j \mathbf{g}_j / N$  ( $\bar{\mathbf{p}} = \sum_j \mathbf{p}_j / N$ ) is the mean genotype (phenotype).

The phenotype of the focal individual after one iteration is given by

$$\mathbf{p}_i^{(1)} = \mathbf{g}_i + \mathbf{\Psi} N \bar{\mathbf{p}}^0 - \mathbf{\Psi} \mathbf{p}_i^0 \quad (\text{S4})$$

$$= \mathbf{g}_i + \mathbf{\Psi} N \bar{\mathbf{g}}^0 - \mathbf{\Psi} \mathbf{g}_i^0. \quad (\text{S5})$$

From (S4) we can express the mean group phenotype after one iteration

$$\bar{\mathbf{p}}^{(1)} = (\mathbf{I} - \mathbf{\Psi} + N\mathbf{\Psi})\bar{\mathbf{g}}. \quad (\text{S6})$$

Again, substituting (S6) to (S1), we can express the phenotype of the focal individual after two iterations as

$$\begin{aligned} \mathbf{p}_i^{(2)} &= (I - \Psi + \Psi\Psi)\mathbf{g}_i + (N\Psi(I - \Psi) + N^2\Psi\Psi - N\Psi\Psi)\bar{\mathbf{g}} \\ &= (I - \Psi + \Psi\Psi)(\bar{\mathbf{g}} + \Delta \mathbf{g}_i) + (N\Psi(I - \Psi) + N^2\Psi\Psi - N\Psi\Psi)\bar{\mathbf{g}} \end{aligned} \quad (\text{S7})$$

which can be written as

$$\mathbf{p}_i^{(2)} = \sum_{k=0}^2 (N-1)^k \Psi^k \bar{\mathbf{g}} + \sum_{k=0}^2 (-\Psi)^k \Delta \mathbf{g}_i \quad (\text{S8})$$

8 where  $\Delta \mathbf{g}_i = \mathbf{g}_i - \bar{\mathbf{g}}$  (also  $\Delta \mathbf{p}_i = \mathbf{p}_i - \bar{\mathbf{p}}$ ) is deviation of the  $i$ -th genotype (phenotype) from the mean.

By induction, we conclude that the individual's phenotype after  $n$  generations is given by

$$\mathbf{p}_i^{(n)} = \sum_{k=0}^n (N-1)^k \Psi^k \bar{\mathbf{g}} + \sum_{k=0}^n (-\Psi)^k \Delta \mathbf{g}_i. \quad (\text{S9})$$

10 Fig. A shows phenotypes calculated in three different ways: iterating equation (S1), using equation (S9) or phenotypic equation inferred by [1] for phenotypes at the equilibrium point. Clearly, trajectories created from equations (S1) 12 and (S9) agree, but they may or may not converge to the stable state solution 14 given by equation (1).

For  $n \rightarrow \infty$ , equation (S9) converges if both of its summands converge. This is the case if both matrices  $(N-1)\Psi$  and  $(-\Psi)$  have a spectral radius  $\rho$  of less than one, i.e., all their eigenvalues  $\lambda$  fulfil  $|\lambda| < 1$ . Since for  $N > 1$   $\rho[(N-1)\Psi] > \rho(\Psi) = \rho(-\Psi)$ , our sufficient condition for the convergence of equation (S9) is  $\rho[(N-1)\Psi] < 1$ . In this case, we obtain

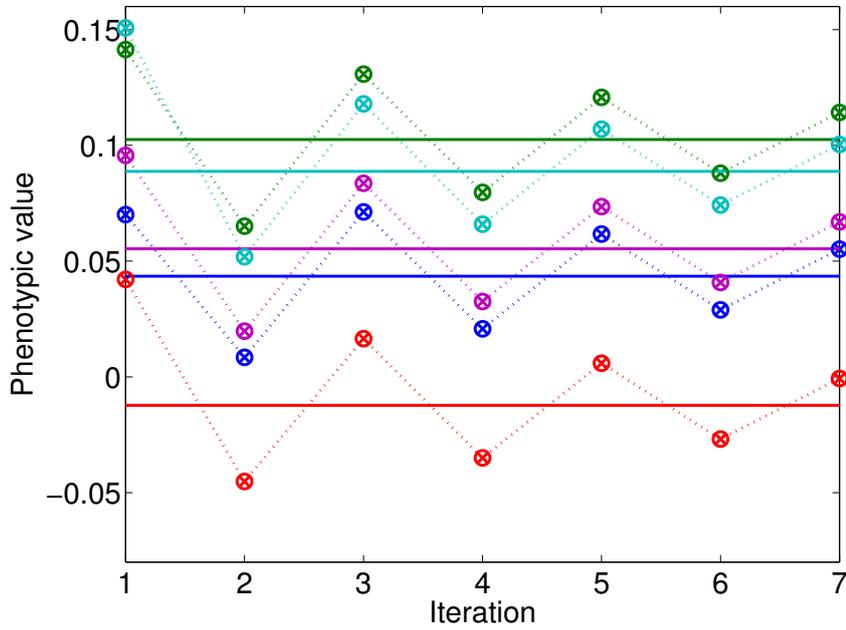
$$\begin{aligned} \mathbf{p}_i^{(\infty)} &= \sum_{k=0}^{\infty} (N-1)^k \Psi^k \bar{\mathbf{g}} + \sum_{k=0}^{\infty} (-\Psi)^k \Delta \mathbf{g}_i. \\ &= (\mathbf{I} - (N-1)\Psi)^{-1} \bar{\mathbf{g}} + (\mathbf{I} + \Psi) \Delta \mathbf{g}_i. \end{aligned} \quad (\text{S10})$$

This solution is equivalent to equation (1), as can be seen after substituting  
16 for  $\Delta g_i$  and  $\bar{g}$  accordingly. Note that even if  $\rho[(N - 1)\Psi] < 1$  is violated,  
expression (S10) is a steady state solution of recursion (S1), however an unstable  
18 one.

## References

- 20 [1] Trubenová B, Hager R (2012) Phenotypic and evolutionary consequences  
of social behaviours: Interactions among individuals affect direct genetic  
22 effects. PLoS ONE 7: e46273.

A



B

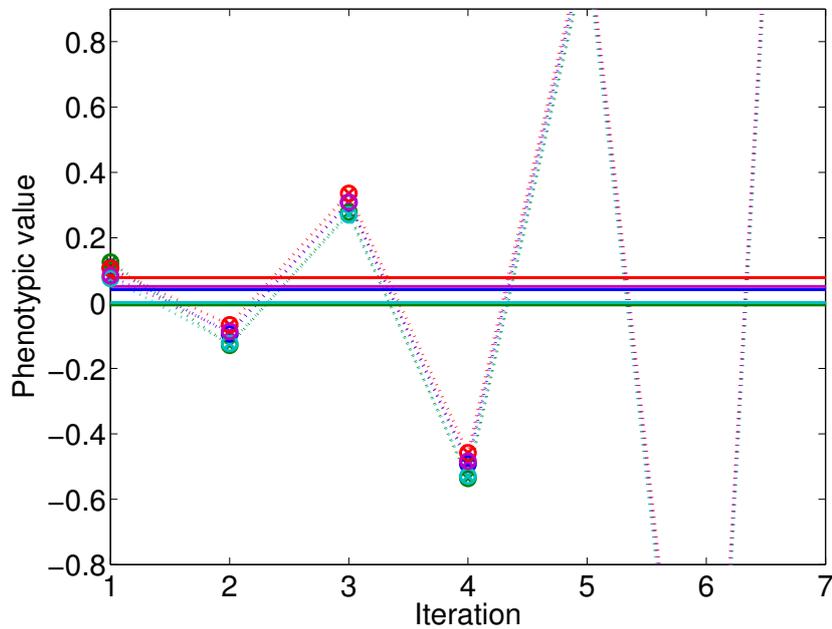


Fig. A: **Comparison of three methods of phenotype calculation.**

The solution calculated using equation (S9) (crosses) agrees with the one obtained by iterating equation (S1) (circles), and may (A) or may not (B) converge to the stable state solution calculated by equation (1) (solid line). Two traits reciprocally influence each other (X and Y) in five interacting individuals. (A)  $\Psi_{12} = \Psi_{21} = 0.2$ . (B)  $\Psi_{12} = \Psi_{21} = 0.5$ . Different colors represent different individuals.